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Why not walk faster?

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Bipedal walking following inverted pendulum mechanics is constrained by two requirements: sufficient kinetic energy for the vault over midstance and sufficient gravity to provide the centripetal acceleration required for the arc of the body about the stance foot. While the acceleration condition identifies a maximum walking speed at a Froude number of 1, empirical observation indicates favoured walk–run transition speeds at a Froude number around 0.5 for birds, humans and humans under manipulated gravity conditions. In this study, I demonstrate that the risk of 'take-off' is greatest at the extremes of stance. This is because before and after kinetic energy is converted to potential, velocities (and so required centripetal accelerations) are highest, while concurrently the component of gravity acting in line with the leg is least. Limitations to the range of walking velocity and stride angle are explored. At walking speeds approaching a Froude number of 1, take-off is only avoidable with very small steps. With realistic limitations on swing-leg frequency, a novel explanation for the walk–run transition at a Froude number of 0.5 is shown.

Keywords: walk; inverted pendulum; run; transition; biped

1. INTRODUCTION

Bipedal walking fits a mechanical description as an 'inverted pendulum' (figure 1), in which the body's kinetic energy turns to potential energy at midstance and is returned as kinetic energy as the body falls during the second half of stance (Cavagna *et al.* 1977); this is the mechanical definition of walking. Alternatively, walking can be defined as locomotion in which at least one foot is always in contact with the ground (Hildebrand 1985). These two definitions for walking can be used to determine two mechanical requirements for walking and may be used to explore speed and leg angle (or step length) conditions under which walking as an inverted pendulum is possible. The mechanical definition of walking puts minimum speed constraints on walking, as there must be sufficient kinetic energy to drive the vault over the slowest and highest position at midstance. The requirement that contact should be maintained with the ground places upper limits on walking speed. As the body vaults over, or around, the foot, it must experience a sufficient acceleration towards the foot (centripetal) to result in the changes in body-heading inherent in travelling around an arc. As the foot does not clamp on to the ground, the leg cannot oppose tension forces. Thus, when the required centripetal acceleration of the body is not provided by gravity, a 'take-off' condition is reached and the inverted pendulum mechanism of walking fails: the foot is either pulled off the ground (first half of stance) or dragged along the ground (second half). This condition for walking has been described (see Alexander 1989), considering either the forces or the accelerations applicable to midstance, using the dimensionless Froude number F_r

$$F_r = \frac{\text{'centrifugal force'}}{\text{weight}} = \frac{mV^2/l}{mg} = \frac{V^2}{gl} < 1, \qquad (1.1)$$

where m is the mass, l the leg length, g the acceleration due to gravity and V the velocity. The observations that bipeds over a wide range of sizes (including birds (Gatesy & Biewener 1991) and humans) and humans under a range of manipulated gravity conditions (Kram *et al.* 1997) all elect to change gait from a walk to a run at $F_r \approx 0.5$ substantiate suggestions that this Froude number represents a general mechanical limitation; however, the limiting value at midstance should be $F_r = 1$. While models of walking incorporating limb compliance (Alexander 1992) can account for this, the required input parameters are somewhat specific. Kinematic factors have been related to the walk–run transition in humans and structural limits or fatigue have been suggested (Minetti *et al.* 1994; Hreljac 1995). However, 'a simple mechanical model that predicts a transition at a Froude number of 0.5 remains elusive' (Kram *et al.* 1997).

In this paper I explore the implications of inverted pendulum mechanics on stance. I follow the extreme simplification of the inverted pendulum using a 'compass gait' model (following Alexander 1977): a point mass vaults over a rigid, massless leg and one leg (and only one leg) maintains contact with the foot placement on the ground at any time—resulting in a duty factor of 0.5. I define the range of velocities and step lengths (or step angles) in which walking can be maintained, and identify the influence of step frequency and swing-leg mechanics as a drive towards walk–run transition at $F_r < 1$.

2. METHODS

The requirement of 'take-off' avoidance, when the gravitational acceleration g acting on the mass no longer exceeds the centripetal acceleration required to keep the limb of length l on the ground, can be expressed at midstance as

$$\frac{V^2}{l} \le g, \qquad (2.1)$$

equivalent to expression (1.1). However, in the case of a simple inverted pendulum the risk of take-off is greater at the limits of stance; indeed, at midstance the foot placement should be most secure (figure 1). This is because (i) the velocity of the mass is higher both before the kinetic energy is converted to potential and after the potential energy has been reconverted to kinetic during the vault and (ii) the component of gravity in line with the leg is reduced when the leg moves from a vertical stance. Giving velocity in terms of angular velocity and leg length ($\omega = V/l$), the non-take-off condition can be expressed for any angle of stance leg from vertical ϕ as

$$\omega^2 l \le g \cos(\phi). \qquad (2.2)$$

Considering an ideal inverted pendulum as the mass vaults over the foot, no mechanical energy ME is lost

$$\text{ME} = \text{PE} + \text{KE} = mgh + \frac{1}{2} I\omega^2 = \text{constant}. \qquad (2.3)$$

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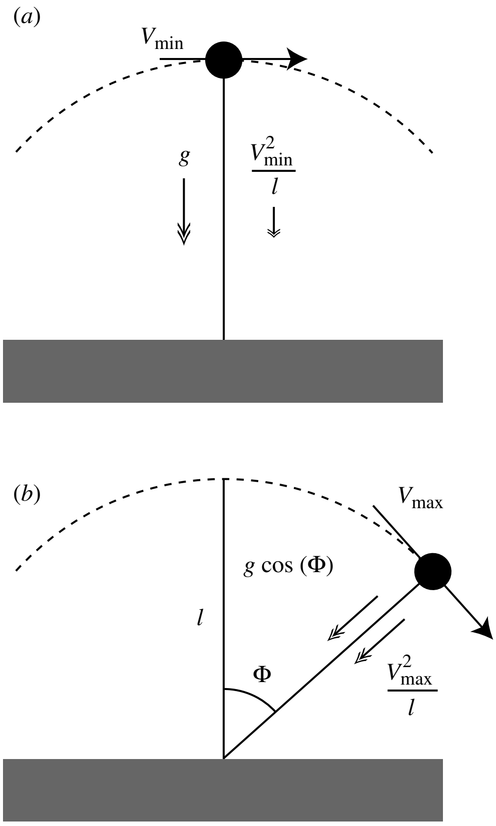

Figure 1. Stance during the 'compass gait' model of walking. The point mass (black circle) vaults over the massless, rigid leg of length l, following an arc about the leg's connection with the ground. The components of acceleration acting in line with the leg are indicated for two positions: (*a*) midstance, or $\phi=0$, when the velocity of the mass is at a minimum for the step (V_{min}) and gravity acts directly in line with the leg and (*b*) at the extreme of leg angle $\phi=-\Phi$ or Φ, when the velocity of the mass is highest for the step (V_{max}) and a smaller component of gravity acts along the leg. The conditions for successful inverted pendulum walking are considered broken if there is insufficient energy for the vault over midstance or if the centripetal acceleration required to keep the leg arcing around the foot cannot be provided by gravity.

Taking the ground as the datum for the potential energy, height h is given as a function of l and ϕ. For a point mass, the second moment of mass I for the body about the foot is ml^2. Thus,

$$\mathrm{ME} = mgl\cos(\phi) + \frac{1}{2}ml^2\omega^2 = \text{constant}, \tag{2.4}$$

and the angular velocity ω_ϕ can be calculated for any value of ϕ by

$$\omega_\phi = \sqrt{\frac{2\left(\frac{\mathrm{ME}}{m} - gl\cos(\phi)\right)}{l^2}}. \tag{2.5}$$

In order to determine the change in time (dt) spent travelling across each small change in leg angle (dϕ), a numerical calculation was made for each leg angle (simple analytical solutions are limited to small angles)

$$\mathrm{d}t = \frac{\mathrm{d}\phi}{\omega_\phi}. \tag{2.6}$$

The period T for the vault from $\phi=-\Phi$ to $\phi=+\Phi$ was calculated as the sum of all dt. The horizontal distance travelled over this period, the step length, is $2l\sin(\Phi)$. The mean horizontal velocity $\overline{V_x}$ was calculated, with the numerical calculation of the step period, from

$$\overline{V_x} = \frac{2l\sin(\Phi)}{T}. \tag{2.7}$$

The inverse of step period is the step frequency f. Given that one and only one leg is in contact with the ground at any time, step frequency also applies to the frequency required for the swing leg. The time taken for an inverted pendulum to travel over the step angle was calculated by the method described above. This must equate to the length of time required for the swing leg to get into place for the next step.

(a) *An analytical limit to stride angle*

An analytical expression can be found for the maximum step angle Φ possible before 'take-off' conditions are exceeded. This angle occurs when the mean horizontal velocity of the walker is as slow as possible but still allows the vault over midstance—when the velocity and kinetic energy at the top of the stance approaches zero. As the inverted pendulum tips over the highest point, the drop in potential energy results in an increase in kinetic energy and velocity (from conservation of mechanical energy)

$$\Delta\mathrm{PE} = mgl(1-\cos(\Phi)) = \frac{1}{2}mV^2. \tag{2.8}$$

Thus, the non-take-off condition loses both length and mass terms

$$\frac{V^2}{l} = 2g(1-\cos(\Phi)) \leq g\cos(\Phi). \tag{2.9}$$

Gravity also cancels; the angle condition simplifies to

$$\Phi \leq \cos^{-1}\left(\frac{2}{3}\right). \tag{2.10}$$

Therefore, the maximum possible step angle for a passive inverted pendulum vault, while avoiding take-off conditions, is 48.2 degrees. This is also the angle (counting 'upright' as 0°) at which a toppling chimney would change from a structure experiencing compression to a structure experiencing tension.

3. RESULTS AND DISCUSSION

The limiting conditions for walking are presented for an inverted pendulum of a leg length of 1 m under gravity of 9.81 m s^{-2} (figure 2*a*). The low speed limit for walking, due to the requirement of sufficient kinetic energy to power the vault over the leg, is indicated by (i): the slowest walking speeds are only achievable with small step lengths and low step frequencies. Beyond 0.56 m s^{-1}, the maximum potential step length reduces and the frequency requirement for any given step length increases. The take-off condition prohibits large step angles at higher speeds both because of the higher velocities at the extremes of stance and the reduced component of gravity in line with the leg. Take-off can be avoided at highest speeds only with small step lengths. However, this necessarily results in high step and swing-leg frequencies. Walking at high Froude numbers and the associated high step frequencies is unappealing to humans. Bertram & Ruina (2001) report step length, speed and step frequency relationships for men of appropriate leg length and find conditions above 2.2 m s^{-1}, 2.5 Hz and 0.9 m step length (figure 2*a*) are avoided for treadmill and across-ground locomotion. The model shows that the relationship between walking step length, frequency and speed is constrained by take-off conditions even at speeds considerably below a Froude number of 1. If humans are unwilling to swing their legs at above 2–3 Hz, they will not be able to achieve inverted-pendulum walking above 2.3–2.6 m s^{-1}.

In order to view these results in a more general context, it is helpful to normalize the values. Mean horizontal velocity thus becomes the non-

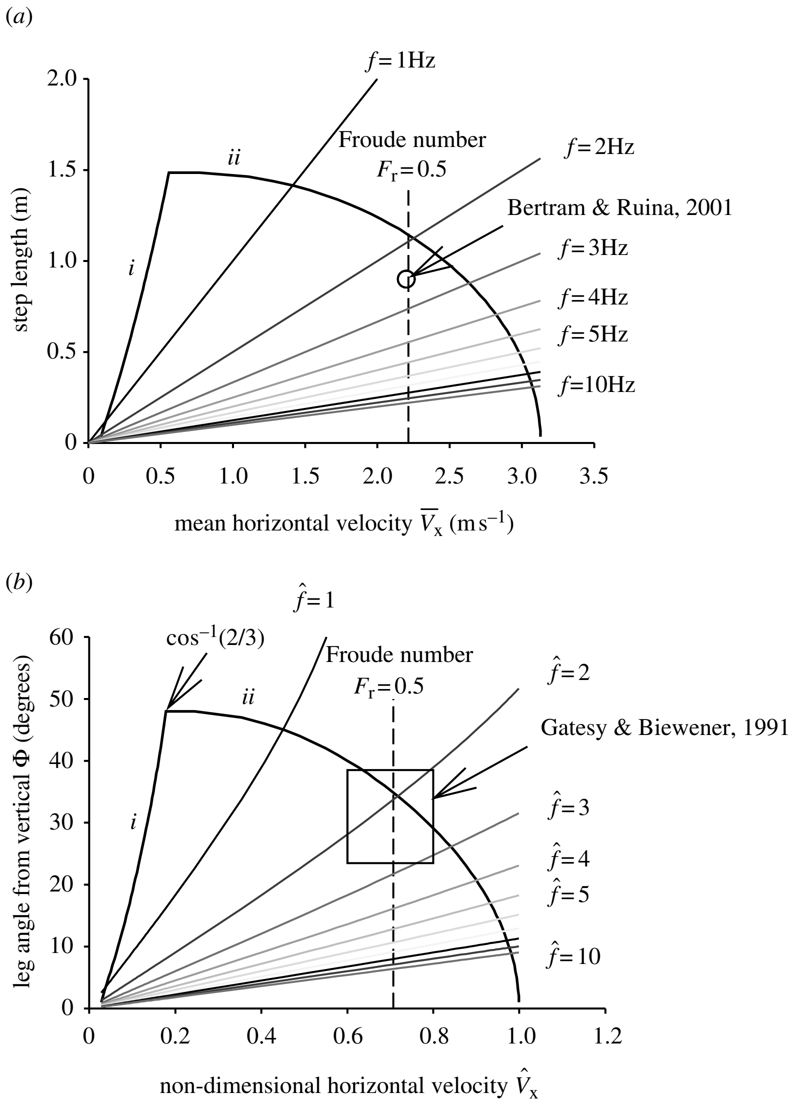

Figure 2. Limiting parameters for walking as an inverted pendulum with sufficient energy for the vault past midstance (i) but without 'take-off' (ii). (*a*) Values appropriate for a human walking on Earth ($l=1$ m, $g=9.81$ m s^{-2}). High walking speeds can only be achieved with low step lengths and high step frequencies. An approximate upper limit to comfortable walking speed for men is indicated by a circle. In non-dimensionalized form (*b*), the step frequency $\hat{f}$ is the step frequency normalized by the frequency of a pendulum of length l swinging over small angles. The rectangle bounds the range of values relating to the walk–run transition for seven species of ground-dwelling bird ranging from 0.045–90 kg (where Φ is taken as half of the stance leg excursion angle published in Gatesy & Biewener (1991)). The largest possible step angle, 48.2°, is indicated but is achievable at only one velocity. Lower leg angles are required for higher walking speeds; however, this requires higher step frequencies. The combination of avoidance of take-off and a limitation to swing-leg frequency provides realistic limitations to inverted pendulum style walking speeds and may account for the consistent preferred speed of the walk–run transition in bipeds of $F_r=0.5$ (vertical dotted lines).

dimensionalized form $\hat{V}_x$

$$\hat{V}_x = \frac{\overline{V_x}}{\sqrt{gl}} = \sqrt{F_r}. \qquad (3.1)$$

This is an alternative form (though resulting in numerically different values) of the Froude number with the benefit of scaling linearly with speed. Thus, the preferred walk–run transition speed for bipeds occurs at a Froude number (expression (1.1)) of around 0.5, or a non-dimensionalized velocity of around 0.707. In both cases, the absolute limit to walking as an inverted pendulum occurs at a value of 1.

Frequency is normalized using the frequency expected for a pendulum of length l swinging over small angles. It is important to note that this is not an accurate model of the swing leg, which swings over large angles, and is connected to a moving and accelerating hip. However, it does provide a useful comparison and is presented to allow a qualitative impression of the frequency of the swing leg compared with approximate passive conditions. The normalized step (or swing) frequency, $\hat{f}$, is given by

$$\hat{f} = f\pi\sqrt{\frac{l}{g}}. \qquad (3.2)$$

This indicates the step or swing frequency as a multiple of the pendulum frequency for a leg of length l.

In order to achieve walking as an inverted pendulum at close to $F_r=1$, the swing leg must recover at many times the passive swing frequency for a pendulum of length l. High step frequencies can be achieved by two means: (i) a functional reduction in the pendulum length of the swing leg by mass-redistribution and (ii) driving of the swing leg by muscular and elastic effort. Pendulum mechanics indicate that reduction in the functional swing-leg length, through evolutionary, developmental or behavioural (including knee and ankle bending) activity, has only a limited potential effect: a doubling of the frequency requires a quartering of the leg length. 'Driving' a swing leg above its passive swing frequency is costly. In terms of kinetic power requirements, the cost of oscillating a mass is proportional to $\Phi^2 f^3$. Thus, independent of size and gravity, similar issues determine the relationship between swing-leg frequency and take-off avoidance.

One implication of this walking model is the identification that foot contact with the ground is least secure at the beginning and end of stance. Thus, the mechanism by which mechanical energy is contributed to the walking system may be influenced by the requirement to maintain good contact with the ground throughout stance. One (but not the only; Kuo 2002) benefit of powering with gastrocnemius activity and plantar flexion (ankle extension) towards the end of stance may be to provide a compressive force (or lengthening of the leg) between the foot and the ground at the instant when, left to passive vaulting, foot–ground contact forces would be at a minimum.

It should be noted that the possibility of walking does not necessarily indicate that it is the most desirable gait for a given speed and that the region indicated as achievable by a vaulting inverted pendulum (figure 2) does not provide strict limits to walking parameters. For instance, race-walkers certainly operate above the velocity predicted, even for the extreme case of $F_r = 1$. However, outside this region the passive mechanism of inverted pendulum KE–PE–KE energy transfer (Cavagna *et al*. 2002) must become compromised and the use of spring-like mechanisms (a running gait) might become energetically favourable. In conclusion, provided deviation from inverted pendulum mechanics and high swing-leg frequencies are both costly, this model indicates a realistic explanation for why the observed walk–run transition speed is well below $F_r = 1$.

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